The Relevance of Interpretation of Probability for Modern Physics

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INTRODUCTION

- Short development of probability and statistics
- Emergence of statistical laws
- Statistical laws and atomism in modern physics
- Statistical laws of nature?
- Modern interpretations of probability
- The combination of dynamic and statistical laws in QM
- Some related problems of probability theory and QM interpretation

DEVELOPMENT OF PROBABILITY & STATISTICS

- The theory of probability emerges in the same period as Newtonian physics (Pascal & Fermat, cca 1660)
- Rooted in games of chance → emergence of theory made possible by:
  - the *regularity* of dices
  - not enough developed algebra
  - modern humanistic curiosity in experimental practices beats moral restrictions
  - Influence of Aristotle‘s belief that a theory of chance is not possible
- Parallel development of the theory of errors – development of the notion of the mean and error
- Bošković → Legendre‘s analiytical method of least squares (1805)
- Galton‘s regression in studies of heredity (1885) mathematically equivalent to the method of least sqares
- „Gauss-Laplace synthesis“ (Stigler): probabilistic concepts enter the theory of errors → theoretically founded error estimation Δx is possible only with the use of probability distributions

EMERGENCE OF STATISTICAL LAWS

- The term statistical law emerged in social sciences
- Quetelet's use of the normal curve in biological & social sciences → „invents" the average man

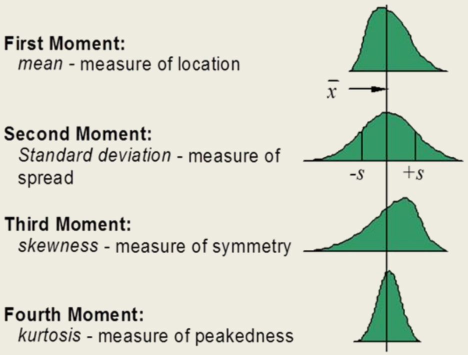
„The greater the number of individuals observed, the more do individual pecularities, whether physical or moral, become effaced, and allow the general facts to predominate, by which society exists and is preserved" (Quetelet, 1835).

„…as if the chests measured had been modeled on the same type, on the same individual, an ideal if you wish, but one whose proportions we can learn from sufficiently prolonged study" (Quetelet, 1846).

- The main shift: Δx acquires the ontological status of real variation within a population

- Karl Pearson introduces the 4 moments of (unimodal) curves, interpreting them as real parameters of populations: mean (*n* = 1), variance (*n* = 2), skewness (*n* = 3), curtosis (*n* = 4)

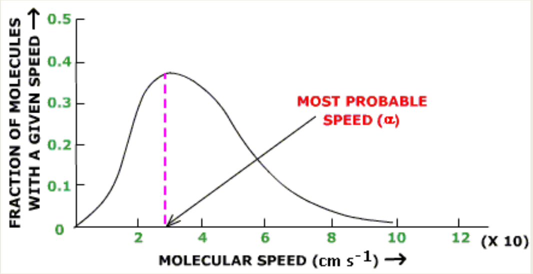
$$M_n = \int_{-\infty}^{\infty} (x-c)^n f(x)\,dx$$



- Pearsonian revolution: distributions not individual measurements are the object ob interest
- Status of probability distributions and their averages – what is „real“: population vs. average
  1. Measurements: average value is real, the rest dismissed as error/deviation
  2. Statistics: population individuals, variance, distribution real; not real population averages (the average man)?
  3. Statistical mechanics: atoms, velocities, individual kinetic energies, temperature (average kinetic energy) all taken as real

STATISTICAL LAWS & ATOMISM

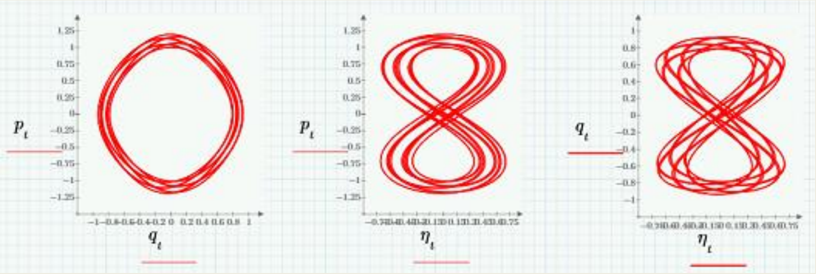
- Establishment of atomism in modern physics inextricably connected with establishment of statistics within physics
- Statistical distributions enter physical theories: Maxwell's law of molecular velocities in a gas (1860)



- Δx is viewed as a real property of a populaton of particles (Quetelet's influence)
- Impossibility of calculating trajectories because of collisions (Maxwell): instabilities/singularities, dynamically unstable system with exponentially diverging trajectories for small variations in initial conditions
- Immediate debates about mechanical vs. statistical views (determinism vs. indeterminism)

- Boltzmann entropy law (1872): introduction of time dependent probability distribution
- Crucial new concepts in statistical physics (Boltzmann):

  1. Ergodicity assumption (argument for calculating with averages): each molecule goes through all states of the phase space and in the limit we get time averages → probability densities as time averages in phase space

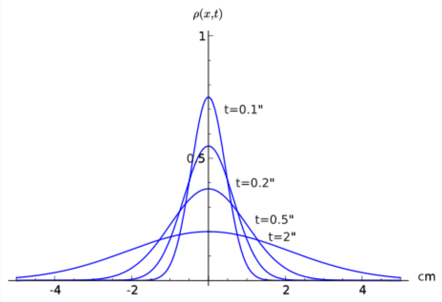


  2. Ensembles: the reason for introducing them is that the independent particles obey a probability distribution *at a single moment*

- This marks the explicit transition of focus from the dynamics of a single entity to multitude
- Multitude as a concept (ensemble of molecules) is explicitly introduced into physical theories proper

- Probability enters classical mechanics: probabilistic solution of Poincare‘s three body problem (1890)
- Development of ergodicity theory tries to found probabilistic laws on supposedly underlying mechanical dynamic laws → mathematical development of measure theoretic probability (probability on a continuous phase space, not in principle countable ensembles)
- Atomic hypothesis gains recognition after Einstein‘s explanation of Brownian motion (1905) by a dynamic law predicting the development of a statistical distribution

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial x^2} \rightarrow \rho(x,t) = \frac{\rho_0}{\sqrt{4\pi D t}} e^{-x^2/4Dt}$$



- Einstein calculates the second moment of the distribution, which is observable (Perrin 1909)

$$\overline{x^2} = 2Dt$$

DYNAMIC VS. STATISTICAL LAWS

- Various attitudes towards laws of nature:
  - Laws tied to deductive systems
  - Laws hold for universals
  - Antirealists: laws do not exist at all (C. S. Peirce), symmetries instead of laws (van Frassen)
  - Antireductionist view
- Laws as central propositions formulated within some scientific theory in scientific language (Weingartner & Mittelstaedt, *Laws of Nature*)

DYNAMIC LAWS

- Uniquely determined states with predecessor states
- Determination of all parts of the system
- Periodicity
- Perturbative stability

Examples:

- Newton's 2^{nd} law
- Newton's gravitational law
- Law of continuity
- Maxwell's equations

STATISTICAL LAWS

- Non uniquely defined states (no continuous temporal development)
- Not all parts of the system are determined → results given in terms of probabilities/averages
- Non-periodicity
- Loss of information (entropy)

Examples:

- Maxwell's law of velocity distribution of particles
- Law of radioactive decay
- Entropy law
- Diffusion law

- What is there lawlike about statistical laws?

Law of large numbers (LLN):

For independent random variables x_i: $\bar{X}_N = \frac{1}{N}\sum_1^N x_i$ converges to a mean value when $N \to \infty$

Central limit theorem (CLT):

Describes the size and the distributional form of the stochastic fluctuations around the mean value during this convergence

- LLN is the mathematical formulation of the recognition that even when with singular events there is a degree of uncertainty in populations/ensembles there is a degree of regularity
- Continuous distributions are idealizations/models achievable in the limit
- LLN & CTL embedded in the frequentist concept of probability
- Statistical laws are implicit to scientific practice: working with distributions, LLN & CLT are the basis of statistical hypothesis testing

MODERN INTERPRETATIONS OF PROBABILITY

- Classical conception = epistemic: we live in a deterministic world and the use of probability is a sign of our ignorance
- Division to subjective (Bayesian) and objective probability explicitly at least from Poisson on

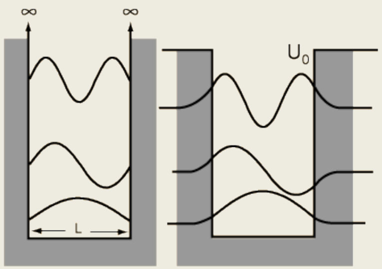
Probability = degree of belief

Probability = limit of relative frequencies

- 20th century:
  - Frequentist approach (von Mises)
  - Logical approach (Keynes)
  - Subjective approach (de Finetti) → decision theory (Savage), game theory
  - Propensity theory (Popper)
- All rest on Kolmogorov‘s axioms of probablity (1933)
- Subjective probability: focusses on single events, stresses conditional probability
- Objective probability: focusses on sequences/collectives, stresses independence of events & convergence of relative frequencies
- Even combining expert judgement, which uses subjective probability, needs „calibration“ (weighing) of experts estimations in order to be useful (Cooke, *Experts in Uncertainty*)

QUANTUM MECHANICS & PROBABILITY

- Uses both kind of laws: temporal evolution of probability distributions („probability flow“)
- The first to make explicit testable predictions about statistical distributions themselves
- „Explosion“ of types of probability distributions → functions that used not to be thought of as distributions (e.g. $\sin^2 x$ for stationary states in an infinite well, orbitals of the H-atom)



- Bohmian QM explicitly joins both dynamic and statistical laws by using ensembles

RELATED PROBLEMS OF QM AND PROBABILITY

1. Many interpretations in QM and probability theory:
   - Conflict between interpretations of objective and subjective character
   - Subjective probability keeps the focus on a single event, which is usual for classical physics → enables the talk about a single particle and wavefunction as „its“ state
2. Fundamental ontological status of wavefunction (→ probability density) real – ensemble not real?
3. Wavefunction collapse – an attempt at keeping the focus on the dynamics of single systems
4. Many worlds interpretation of QM (and the multiverse) – establishing as ontologically real the phase space, projection of multiplicity, primarily introduced as ensembles (basically atoms!), to worlds
5. Uncertainty – unavoidably connected with distributions
   - Theory of errors: error/deviation from real value
   - Statistical mechanics: real property as part of distribution
   - QM – Heisenberg uncertainty principle: real property of a single particle?
6. Causality
   - Probability distributions are indications of propensties, *complementary* to causation
   - Statistical laws do not exclude the classical concept of causation
7. Discreteness of phase space in probability – quantization in QM

CONCLUSIONS

- Statistics and probability must deal with ensembles/multiple events to be empirically relevant
- Statistical and probability distributions are models to which empirical events are related via the LLN
- Conflict between singular and multiple events is inherent in probability theory
- The affirmation of atomism in modern physics went hand in hand with the introduction of statistics and the concept of multiplicity in physical theories
- Quantified uncertainty is a property that can meaningfully be conceived of only in relation to statistical distributions
- Bohmian QM is conceptually compatible with these conclusions