Classical Logic vs. Quantum Logic

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Where to start?

When comparing Classical Logic and Quantum Logic, the first step is defining Classical Logic, or asking:

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What does the expression "Classical Logic" make you think of?

Classical Logic

Classical Logic Properties:

- Binarity,
- Commutativity,
- Distributivity,
- Principle of excluded middle,
- Principle of non-contradiction.

Classical Logic Properties

- Binarity:
  Set of truth values - $\mathcal{V}(P) \in \{0, 1\}$
- Commutativity:
  $P \cap Q \equiv Q \cap P$, $P \cup Q \equiv Q \cup P$
- Distributivity:
  $P \cap (Q \cup R) \equiv (P \cap Q) \cup (P \cap R)$
  $P \cup (Q \cap R) \equiv (P \cup Q) \cap (P \cup R)$
- Principle of excluded middle:
  $\mathcal{V}(P \cup \neg P) = 1$
- Principle of non-contradiction:
  $\mathcal{V}(\neg(P \cap \neg P)) = 1$, alternatively $\mathcal{V}(P \cap \neg P) = 0$

Quantum Logic Definition

Two main types:

- Birkoff-von Neumann (B-vN) Quantum Logic
- Fuzzy Quantum Logic

Birkhoff-von Neumann (B-vN) Quantum Logic

- In 1936 Birkoff and von Neumann wrote the article "The Logic of Quantum Mechanics".
- Birkoff and von Neumann wanted to find the logical structure in quantum mechanics which did not conform to classical logic.
- B-vN quantum logic is, among other things, a binary, non-distributive and non-commutative lattice.

Fuzzy Quantum Logic

- Jaroslaw Pykacz used the fuzzy sets idea to build Quantum Logic.
- Fuzzy Quantum Logic is, defined by Pykacz, partial and infinite - valued ($\mathcal{V}(P) \in [0,1]$) which is the connection to a probablistic interpretation of quantum mechanics.
- With Fuzzy Quantum Logic we can analyze non - tested experimental sentences.
- There are generally two sets of set operations used in Fuzzy Quantum Logic:

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- There are generally two sets of set operations used in Fuzzy Quantum Logic:
  - Zadeh's set operations
  - Giles' set operations

Zadeh's Set Operations

- Lotfi A. Zadeh developed the fuzzy set theory.
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- Complement: $\mathcal{V}(P') = 1 - \mathcal{V}(P)$
- Intersection: $\mathcal{V}(P \cap Q) = min[\mathcal{V}(P), \mathcal{V}(Q)]$
- Union: $\mathcal{V}(P \cup Q) = max[\mathcal{V}(P), \mathcal{V}(Q)]$

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- Complement: $\mathcal{V}(P') = 1 - \mathcal{V}(P)$
- Intersection: $\mathcal{V}(P \cap Q) = max[\mathcal{V}(P) + \mathcal{V}(Q) - 1, 0]$
- Union: $\mathcal{V}(P \cup Q) = min[\mathcal{V}(P) + \mathcal{V}(Q), 1]$

Tableau of Comparison

Let's compare different logics considering given properties.

Properties\Logic	CL	B-vN	Z'sFL	Gs'FL
Binarity	$\surd$	$\surd$	$\times$	$\times$
Commutativity	$\surd$	$\times$	$\surd$	$\surd$
Distributivity	$\surd$	$\times$	$\surd$	$\times$
Excluded Middle	$\surd$	$\surd$	$\times$	$\surd$
Non-Contradiction	$\surd$	$\surd$	$\times$	$\surd$

Distributivity or Excluded Middle?

So the question is:

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So the question is:

Which one is more important for quantum mechanics: Distributivity or Excluded Middle?