Galilean Relativity and the Lorentz Contraction

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Split, July 7, 2014

What is Galilean Relativity?

- In honor of the 450th anniversary of Galileo's birth, it seems appropriate to review and appreciate the principle of "Galilean Relativity" or "the equivalence of all inertial frames", since it is a physical principle of enduring importance.
- It is also somewhat obscure, and often misunderstood.

One Worry

- It is sometime said that the Special Theory of Relativity can be derived from two principles:
- 1) Galilean Relativity
- 2) The Constancy of the Speed of Light
- But if so, then General Relativity must *reject* one of these principles, since it is not Special Relativity.
- Which one?

Another Worry

- If the principle is “all inertial frames are equivalent”, then we have to know what an inertial frame is.
- It is also essential to know what sort of “equivalence” is meant: just equivalence of *observable behavior* or equivalence in some deeper sense.

For Example

- In one sense, Newtonian Mechanics is supposed to exemplify Galilean Relativity.
- Indeed, Corollary V in the *Principia* is supposed to prove some sort of relativity principle, at least for impact forces.
- But according to Newton, not all “inertial frames” are physically equivalent: there is exactly one frame at “absolute rest”.

Symmetry Principles

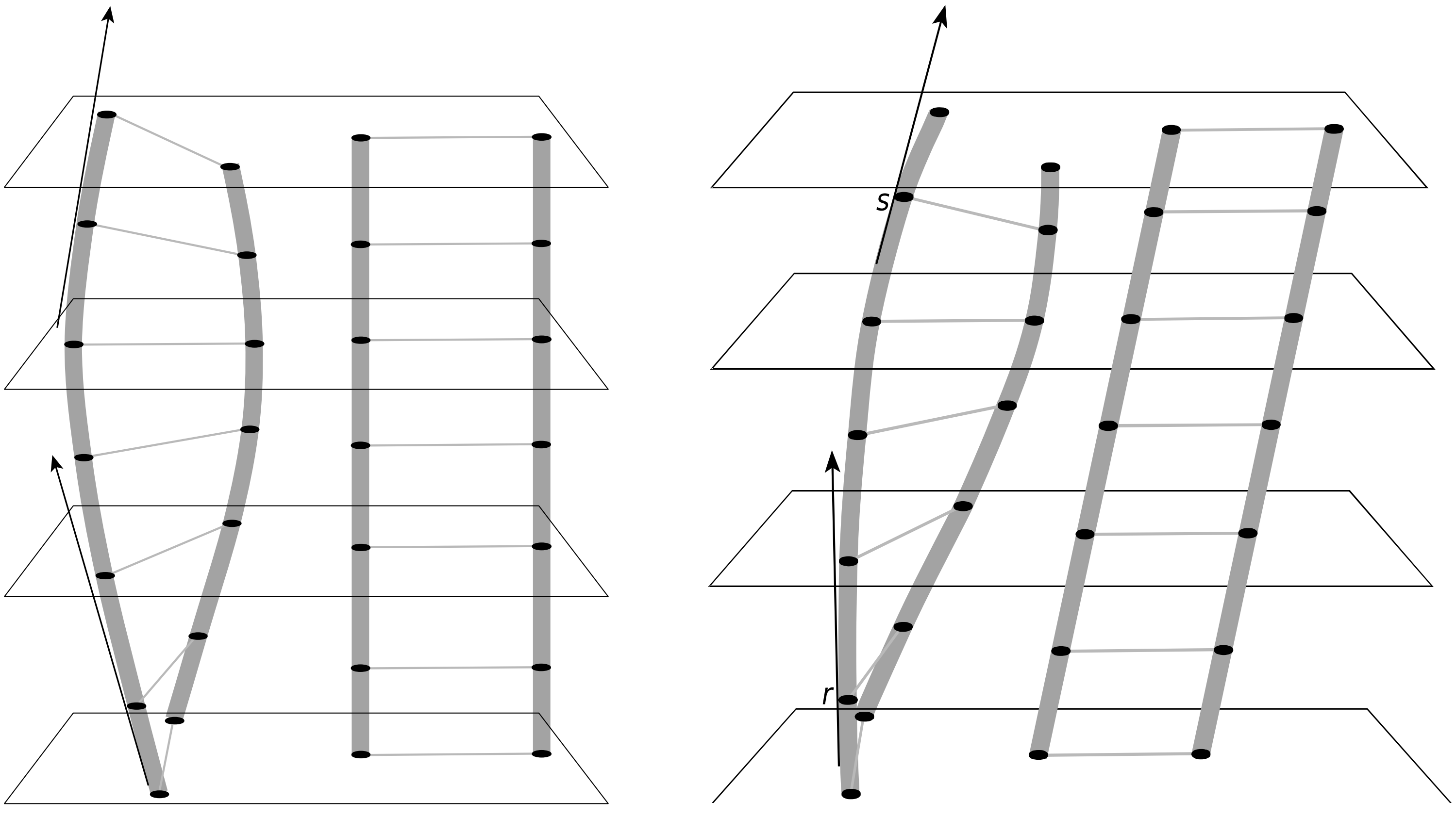
- One way to characterize Galilean Relativity is as an *assertion of the existence of a space-time symmetry*.
- The relevant symmetry would be symmetry under a “boost” of some sort.
- This approach suggests a characteristic that could hold in Newtonian Absolute Space and Time, in Neo-Newtonian Space-Time, and in Minkowski space-time.

Note

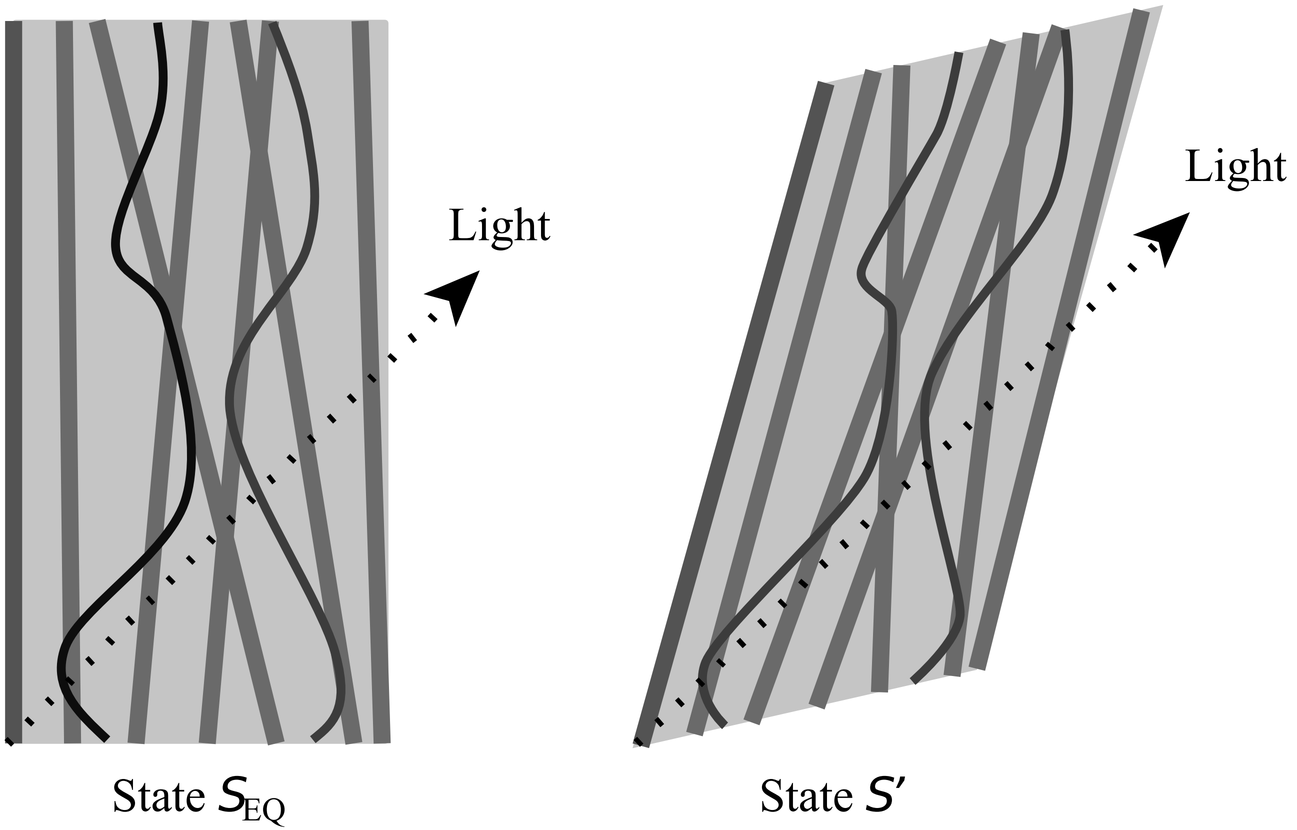
- Insofar as the principle invokes situations that are *observationally indistinguishable*, the symmetry need not be a complete symmetry. This is true in Newtonian Absolute Space. But whatever is not invariant under the symmetry (e.g. Absolute Velocity) cannot be observable or have observable effects. So one would be tempted to drop it, if possible.

- For Newton, only relative distances are observable and only relative velocities appear in the force laws, and these are preserved under the boost.

Classical Boost



Relativistic Boost



Active and Passive Transformations

- In the Relativistic case, the boost creates what Lorentz called “corresponding states”, i.e. states that take the same *coordinate dependent form* relative to different Lorentz frames.
- When both states are then referred to the same frame, this called an “active transformation”.
- Mathematically, though, this seems to be the same procedure as describing the *same* situation using *different* frames of reference.
- But if such a passive transformation is really equivalent to an active transformation (In some sense), then it is *trivial* that there are no observable differences.

Galileo’s Experiment

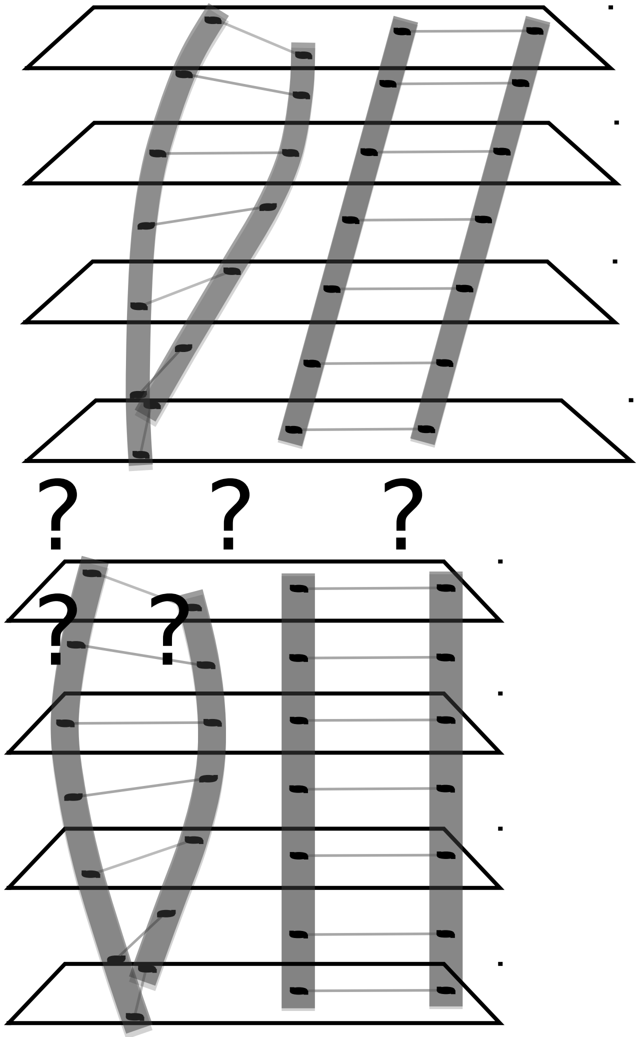
- For a final indication of the nullity of the experiments brought forth, this seems to me a place to show you a way to test them all very easily. Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall in the vessel beneath; and in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction.

Experiment Con't.

- When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow, even though the ship is moving quite rapidly, despite the fact that during the time you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air.

Not a Boost in Above Sense

- Galileo's observations do not compare a given state to a boosted state in the sense we have defined.
- Rather, Galileo discusses observations made *using the same equipment* before and after it has experienced an *acceleration*.
- Nothing in our definition of a boost requires any consideration of the physical effects of accelerating a system!



What Determines the Transition?

- The mathematical boot operation determines the relation between a given *global* state and the corresponding boosted state. But it is rather *the physics of the material* and the *details of the acceleration* that determines what the upper state will be given the lower state.
- Galileo's assertion is that that physical transformation will *lead to* an empirically indistinguishable situation.
- Note: it need not be indistinguishable during the transition!
- What must the physics be like to achieve that?

Corollary VI

- In the *Principia*, after proving Corollary V, Newton proves Corollary VI: *If bodies, any how moved among themselves, are urged in the direction of parallel lines by equal accelerative forces; they will all continue to move among themselves, after the same manner as if they had been urged by no such forces.*

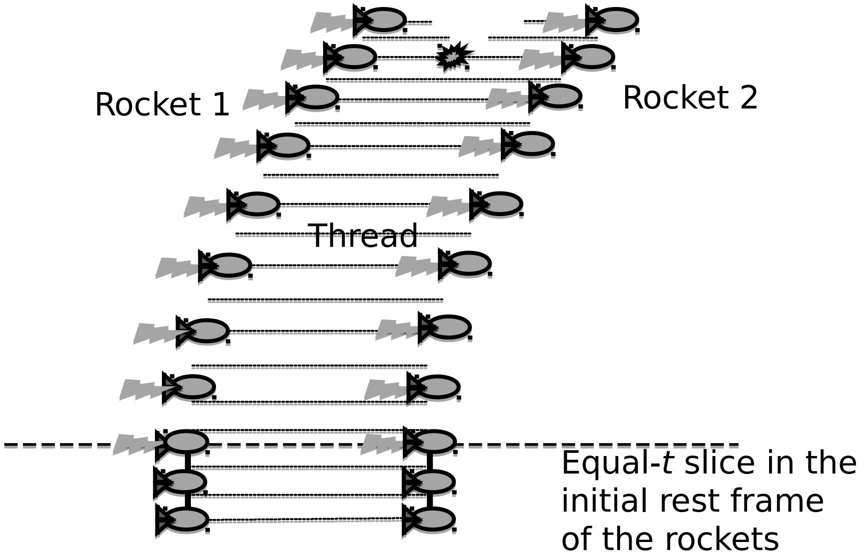
- If Corollary VI holds, then one could fill in the gap between the two states with equal accelerative forces.

- The result would be a Galilean transformation between states, *not a Lorentz transformation*.

The Rocket Puzzle

- In his paper on how to teach Relativity, Bell relates a discussion at CERN about an old puzzle: if a string is tied between projections coming off two identical rockets, and the rockets are ignited simultaneously (in their initial rest frame), so their trajectories will be parallel (i.e. they maintain the same distance apart in the original frame), will the string break?
- (We could add lots of little rockets, one for each little stretch of string. Then it would fit Newton's condition.)

The Set-up



Rocket 1
Rocket 2
Thread
Equal-*t* slice in the
initial rest frame
of the rockets

The Upshot

- Since Newton’s Corollary VI leads to a Galilean transformation as a boost symmetry, and the Lorentz transformation is not the Galilean transformation, Newton’s Corollary VI had better be broken by the Relativistic dynamics implementing Galileo’s experiment.

- In short, the particles in the string must *not* continue to move among themselves in the same way when accelerated as when unaccelerated if Galileo’s phenomenology is preserved.

The Lorentz-FitzGerald Contraction

- The physical effect of acceleration needed to get the right result for Galileo is produced by the physical forces that maintain a “rigid” solid in its equilibrium configuration.

- For example, in order for the pair of rockets to end up in the proper Lorentz-transformed state after the rockets stop firing, they must be *drawn closer together.* If the string were a very strong cable, the rockets would be drawn together by tension in the cable. If the force needed to keep the rockets together is greater than the tensile strength, the cable will break.

The Physics

- What Bell shows in his paper is that the electromagnetic forces in such a "rigid" solid (in his analysis, a single atom) will produce such a "contraction". Absent such forces, the end state after the acceleration would not be the "corresponding state", and Galileo's phenomenology will fail.

- Suppose, for example, the rockets are sending radar signals back and forth. If the cable breaks, and the rockets are not drawn together, then after the acceleration the timing of the signals will change, contrary to Galileo's prediction.

Solids and Equilibrium

- It is exactly because the experimental apparatus is a *solid* that one does not have to concern oneself with the details of the acceleration phase: any part of the instrument can be acted on directly to produce the acceleration, and the internal dynamics of the solid to yield a unique state *once the acceleration is over and the system relaxes back into equilibrium*.
- If the experimental apparatus were not solid, then the final state would depend sensitively on how the acceleration is carried out, and a corresponding state would not, in general, result.

Michelson-Morley

- The Michelson-Morley experiment centrally involves an acceleration: the rotation of the instrument from one orientation to another.
- The apparatus was an elastic solid, so the end state would be a corresponding state, and the interference bands would not shift.
- This prediction *depends critically on the Lorentz-FitzGerald "contraction" as we have defined it*, i.e. on the internal electro-magnetic forces in the apparatus.

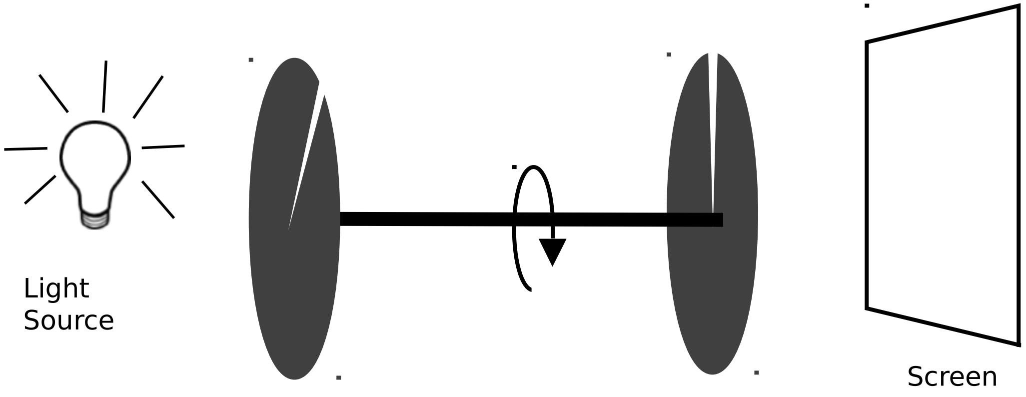
On “Contraction”

- The internal dynamics of the string (breaking) or the cable (deflecting the rockets) or the Michelson-Morley apparatus when rotated must be taken into account when making phenomenological predictions. *In some frames of reference* these effects would be described as a “contraction”. In others, that word would not be useful. But in *every* frame, the internal dynamics makes a difference.
- Without it, the Galilean phenomenology would not occur.

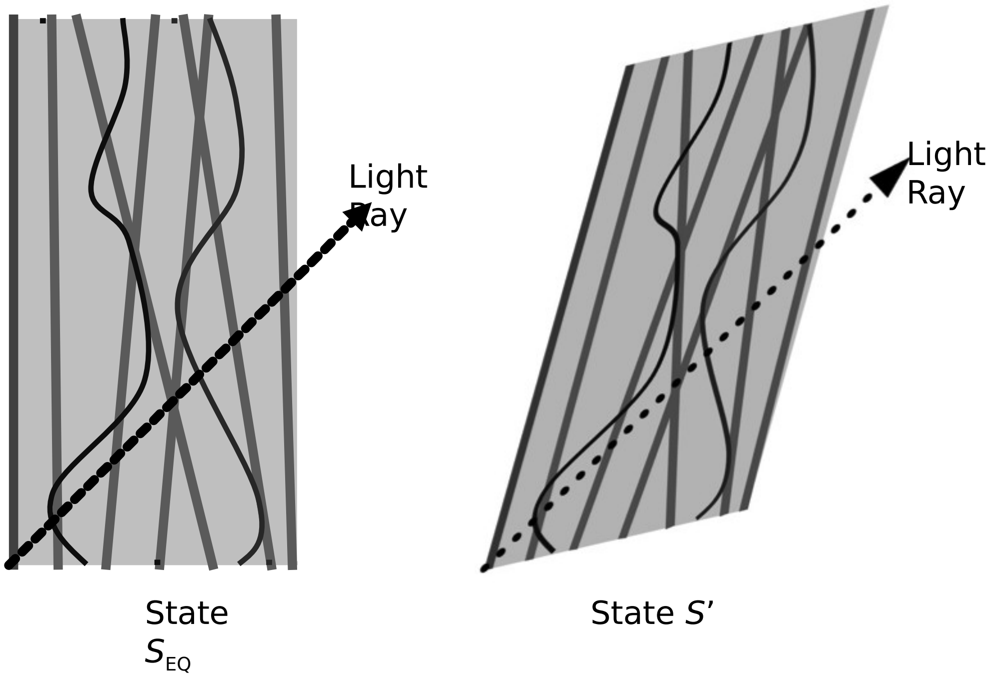
Galilean Relativity and the “Constancy of the Speed of Light”

- The fact that certain phenomena will be invariant before and after an acceleration, together with the existence of a light-cone structure, accounts for “the constancy of the speed of light”.
- Any experimental method designed to be sensitive to the “speed of light” must be sensitive to the trajectory of a light ray through space-time.
- Such a method need not even attempt to *quantify* the “speed of light”.

For Example



Corresponding States



Therefore

- If the machine is tuned to allow the light ray through when oriented in one direction, then rotated or linearly accelerated and allowed to return to equilibrium, the light ray will still go through.

- In this example the light source itself is part of the apparatus, and gets accelerated to a new state.

In Addition….

- The existence of a light-cone structure entails that the trajectory of a light ray is independent of the state of motion of the source, so one would get the same result for light from any source.
- This is all we mean by “the speed of light is constant”.
- Galilean Relativity plus the light-cone structure yields all the phenomena we associate with “the constancy of the speed of light”, even when no speed is assigned to light at all.

In Conclusion

- In Relativity, unlike Newtonian physics, the phenomenon of Galilean Relativity depends on certain dynamical features of the forces that make “rigid” bodies “rigid”.
- These internal forces create changes in the state of an accelerated body that can be reasonably called a Lorentz contraction.
- So the Lorentz contraction, as a *physical* effect, is indeed essential to explain the basic phenomenology of Relativistic physics.